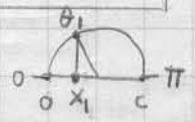
UE Fluids Problem 3+4 Solution Fall '08

Anderson 4.6

$$\frac{z}{c} = \begin{cases} 0.25\left[0.8\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] \\ 0.111\left[0.2 + 0.8\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] \end{cases} \qquad \boxed{\frac{dz}{dx} = \begin{cases} 0.25\left[0.8 - 2\frac{x}{c}\right] & ; \ 0 \le \frac{x}{c} \le 0.4 \\ 0.111\left[0.8 - 2\frac{x}{c}\right] & ; \ 0.4 \le \frac{x}{c} \le 1 \end{cases}}$$

Fourier Analysis: Must do integrals in 2 pieces:



$$\int_0^{\pi}(\,)\,d\theta = \int_0^{\theta_1}(\,)\,d\theta + \int_{\theta_1}^{\pi}(\,)\,d\theta \quad ; \quad \frac{x_1}{c} = \frac{1}{2}(1-\cos\theta_1) = 0.4 \rightarrow \theta_1 = 78.46^\circ = 1.369 \text{ rad}$$

Make trig substitution:

$$\boxed{\frac{dz}{dx}(\theta) = \begin{cases} 0.25\left[0.8 - (1-\cos\theta)\right] = 0.25\left[\cos\theta - 0.2\right] \\ 0.111\left[0.8 - (1-\cos\theta)\right] = 0.111\left[\cos\theta - 0.2\right] \end{cases}}$$

$$\pi A_0 = \int_0^{\theta_1}\left(\alpha - \frac{dz}{dx}\right)d\theta + \int_{\theta_1}^{\pi}\left(\alpha - \frac{dz}{dx}\right)d\theta = \alpha\cdot\pi - \int_0^{\theta_1} 0.25\left[\cos\theta - 0.2\right]d\theta - \int_{\theta_1}^{\pi} 0.111\left[\cos\theta - 0.2\right]d\theta$$

$$= \alpha\pi + 0.05\,\theta_1 - 0.25\sin\theta\Big|_0^{\theta_1} + 0.0222(\pi - \theta_1) - 0.111\sin\theta\Big|_{\theta_1}^{\pi}$$

$$\boxed{A_0 = \alpha - 0.00903}$$

$$-\frac{\pi}{2}A_1 = \int_0^{\theta_1}\left(\alpha - \frac{dz}{dx}\right)\cos\theta\,d\theta + \int_{\theta_1}^{\pi}\left(\alpha - \frac{dz}{dx}\right)\cos\theta\,d\theta = \int_0^{\pi}\alpha\cos\theta\,d\theta - \int_0^{\theta_1} 0.25\left[\cos\theta - 0.2\right]\cos\theta\,d\theta$$

$$- \int_{\theta_1}^{\pi} 0.111\left[\cos\theta - 0.2\right]\cos\theta\,d\theta$$

$$= -0.2559 \quad \text{(numerically)}$$

$$\therefore \boxed{A_1 = 0.1629}$$

$$-\frac{\pi}{2}A_2 = \int_0^{\theta_1}\left(\alpha - \frac{dz}{dx}\right)\cos 2\theta\,d\theta + \int_{\theta_1}^{\pi}\left(\alpha - \frac{dz}{dx}\right)\cos 2\theta\,d\theta = \int_0^{\pi}\alpha\cos 2\theta\,d\theta - \int_0^{\theta_1} 0.25\left[\cos\theta - 0.2\right]\cos 2\theta\,d\theta$$

$$- \int_{\theta_1}^{\pi} 0.111\left[\cos\theta - 0.2\right]\cos 2\theta\,d\theta$$

$$= -0.0436$$

$$\boxed{A_2 = 0.0277}$$

$$C_\ell = 2\pi(\alpha - \alpha_{L=0}) = 2\pi A_0 + \pi A_1$$

$$\therefore \alpha_{L=0} = \alpha - A_0 - \frac{1}{2}A_1 = 0.00903 - \tfrac{1}{2}0.1629 = -0.0724 \text{ rad} = -4.15^\circ$$

For $\alpha = 3^\circ = 0.0524$ rad $\rightarrow$ $\boxed{C_\ell = 2\pi(\alpha - \alpha_{L=0}) = 0.7839}$

Anderson 4.7: $\boxed{C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = -0.1062}$

$$\boxed{x_{cp}/c = \frac{1}{4} - \frac{C_{mc/4}}{C_\ell} = \frac{1}{4} - \frac{-0.1062}{0.7839} = 0.3855}$$

Can check using NACA 4402 airfoil in XFOIL (same camber line as 4412 airfoil, but thinner)